Powers & Polynomials Worksheet

Name ____________________

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1. Find the derivatives of the given functions.

   (a) $y = x^{12}$
   Solution. $y' = 12x^{11}$ $\square$

   (b) $y = x^{-12}$
   Solution. $y' = -12x^{-13}$ $\square$

   (c) $y = x^{4/3}$
   Solution. $y' = \frac{4}{x}x^{1/3}$ $\square$

   (d) $f(r) = \frac{1}{r^{7/2}}$
   Solution. $f(r)$ can be rewritten as $f(r) = r^{-7/2}$.
   Then $f'(r) = -\frac{7}{2}r^{-9/2} = -\frac{7}{2r^{9/2}}$. $\square$

   (e) $f(x) = \sqrt{\frac{1}{x^3}}$
   Solution. $f(x)$ can be rewritten as $f(x) = x^{-3/2}$.
   Then $f'(x) = -\frac{3}{2}x^{-5/2} = -\frac{3}{2x^{5/2}}$. $\square$

   (f) $f(t) = 3t^2 - 4t + 1$
   Solution. $f'(t) = 6t - 4$ $\square$

   (g) $y = 4x^{3/2} - 5x^{1/2}$
   Solution. $y' = 6\sqrt{x} - \frac{5}{2\sqrt{x}}$ $\square$

   (h) $y = 3t^5 - 5\sqrt{t} + \frac{7}{t}$
   Solution. $y' = 15t^4 - \frac{5}{2\sqrt{t}} - \frac{7}{t^2}$ $\square$

   (i) $y = t^{3/2}(2 + \sqrt{t})$
   Solution. y can be rewritten as $y = 2t^{3/2} + t^2$.
   Then $y' = 3\sqrt{t} + 2t$. $\square$

   (j) $y = \frac{x^2+1}{x}$
   Solution. y can be rewritten as $y = x + x^{-1}$.
   Then $y' = 1 - \frac{1}{x^2}$. $\square$

(k) $g(x) = x^{\pi} - x^{-\pi}$

Solution. $g'(x) = \pi x^{\pi-1} + \pi x^{-\pi-1}$. □

2. Find the equation of the tangent line to $f(x) = x^3 + x$ at the point where $x = 2$.

Solution. The slope of the tangent line is given by $f'(2)$. Since $f'(x) = 3x^2 + 1$, $f'(2) = 13$. The point where the tangent line intersects the curve is at $(2, f(2))$, i.e., $(2, 10)$. Using point-slope form, we get $y - 10 = 13(x - 2)$. Simplification yields, $y = 13x - 16$. □